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A METHOD FOR CALCULATING WAVE PACKET TRAJECTORIES AND WAVE HEIGHT--ETC(U)

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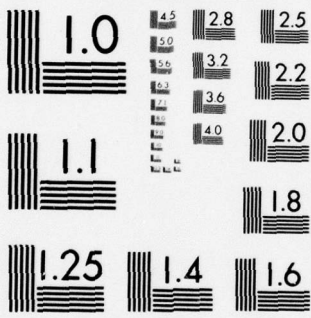
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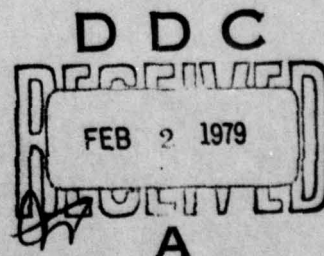
A METHOD FOR CALCULATING WAVE PACKET TRAJECTORIES AND WAVE HEIGHTS: PART II

by J. Ernest Breeding, Jr.

TECHNICAL REPORT NO. JEB-4
Department of Oceanography
Florida State University

November, 1978

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⑨ Technical Rept.

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37 p.

AT- NR

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ABSTRACT

In Part I a method for calculating wave packet trajectories and wave heights is based on the assumption that the water depth contours are locally parallel in the vicinity of each ray point. This method is extended in order to predict the modification to surface gravity water waves in shoaling water when the water depth contours are not parallel. The calculations are greatly simplified by choosing a coordinate system at each ray point in which one axis is aligned parallel with the direction of the gradient of the water depth. Example printouts and plots are presented to illustrate the wave prediction method. It is discovered that when waves initially approach sinuous water depth contours symmetrically with respect to the beach there can be more energy in the bays than at the headlands.

ACKNOWLEDGMENTS

I thank Shelley Horton for her assistance in preparing the computer program and figures in this report. The research was supported by the Geography Programs, Earth Sciences Division, Office of Naval Research under contract No. N00014-77-C-0329.

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CHAPTER I INTRODUCTION

This is a supplement to the report by Breeding, J. Ernest, Jr., K.C. Matson, and Nouroollah Riahi, "A Method for Calculating Wave Packet Trajectories and Wave Heights," Department of Oceanography, Florida State University, Tallahassee, Report No. JEB-1 (1978) hereafter referred to as Part I. The program for predicting the modification to waves in shoaling water described in Part I is developed on the assumption that the water depth contours are locally parallel in the vicinity of each ray point. This method results in accurate wave predictions for many examples of bottom topography. However, the wave height predictions are not accurate when the water depth contours are highly non-parallel in the region of a ray point.

The objective in this report is to extend Part I in order to make accurate wave predictions when the water depth contours are not parallel. The theory is developed in Chapter II. The calculations are greatly simplified by making them in a $x'y'$ -coordinate system which is chosen so that at each ray point the positive x' -axis is in the direction of the gradient of the water depth. As a result, the first partial derivatives of the water depth and wave velocities with respect to y' vanish. Further, there is a simplification in the second partial derivatives involving y' . In Chapter III modifications to the computer program are described. To illustrate the wave prediction program two examples of nonparallel water depth contours are presented in Chapter IV. An interesting result is that when waves initially approach sinuous water depth contours symmetrically with respect to the beach (headon) it is possible for there to be more energy in the bays than at the headlands. This is in contrast to the result for monochromatic trajectories where under similar conditions there is always more energy predicted at the headlands than in the bays.

CHAPTER II THEORY FOR NONPARALLEL WATER DEPTH CONTOURS

2.1 Spatial Derivatives of the Water Depth. At each point of a wave packet trajectory the calculations are made in a $x'y'$ -coordinate system where the x' -axis is taken in the direction of the gradient of the water depth. The particulars of a wave packet trajectory are tabulated in a xy -coordinate system which retains a fixed orientation with respect to the water depth grid. The relationships between these coordinate systems and a specific ray point for a set of nonparallel water depth contours are shown in Figure (2-1). Equations relating these coordinate systems are given by

$$x' = x \cos \alpha + y \sin \alpha \quad (2-1)$$

$$y' = -x \sin \alpha + y \cos \alpha \quad (2-2)$$

$$\tan \alpha = \frac{\partial h}{\partial y} / \frac{\partial h}{\partial x} \quad (2-3)$$

where α is the angle by which the x' -axis is rotated with respect to the x -axis and h is the water depth.

The partial derivatives of h in the $x'y'$ -coordinate system with respect to the partial derivatives of h in the xy -coordinate system are given by

$$\frac{\partial h}{\partial x'} = \frac{\partial h}{\partial x} \cos \alpha + \frac{\partial h}{\partial y} \sin \alpha \quad (2-4)$$

$$\frac{\partial h}{\partial y'} = -\frac{\partial h}{\partial x} \sin \alpha + \frac{\partial h}{\partial y} \cos \alpha = 0 \quad (2-5)$$

$$\frac{\partial^2 h}{(\partial x')^2} = \frac{\partial^2 h}{\partial x^2} \cos^2 \alpha + 2 \frac{\partial^2 h}{\partial x \partial y} \sin \alpha \cos \alpha + \frac{\partial^2 h}{\partial y^2} \sin^2 \alpha \quad (2-6)$$

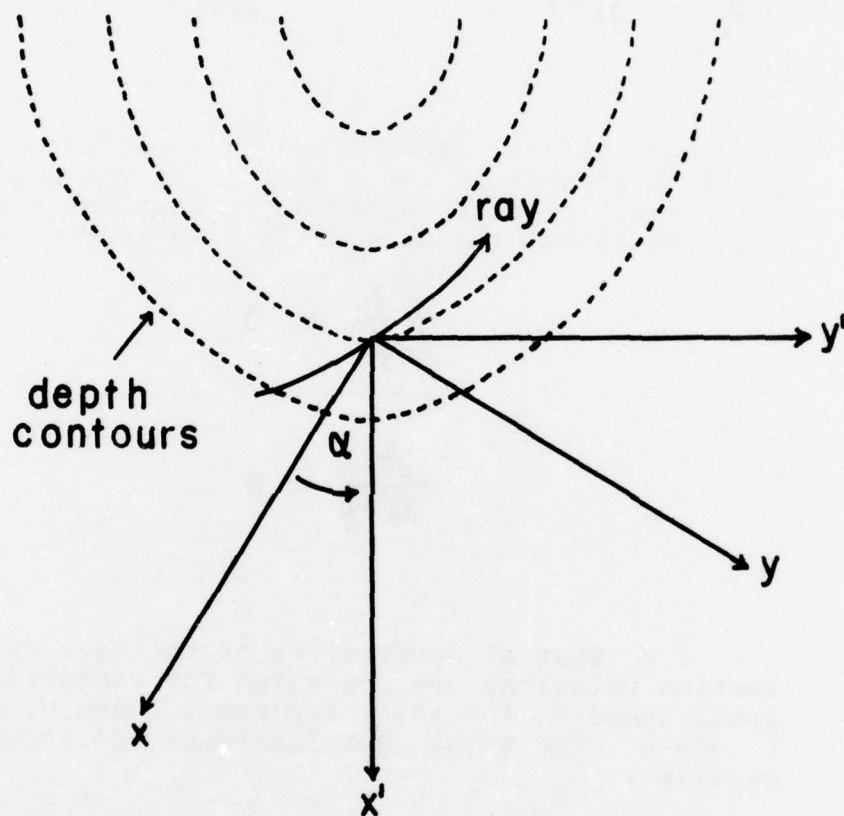


Figure (2-1). Relationships between the coordinate systems and the water depth contours.

$$\frac{\partial^2 h}{(\partial y')^2} = \frac{\partial^2 h}{\partial x^2} \sin^2 \alpha - 2 \frac{\partial^2 h}{\partial x \partial y} \sin \alpha \cos \alpha + \frac{\partial^2 h}{\partial y^2} \cos^2 \alpha \quad (2-7)$$

$$\frac{\partial^2 h}{\partial x' \partial y'} = \left(\frac{\partial^2 h}{\partial y^2} - \frac{\partial^2 h}{\partial x^2} \right) \sin \alpha \cos \alpha + \frac{\partial^2 h}{\partial x \partial y} (\cos^2 \alpha - \sin^2 \alpha) \quad (2-8)$$

In Equation (2-5), $\partial h / \partial y' = 0$ as a result of choosing the positive x' -axis in the direction of the gradient of the water depth. If the water depth contours are locally parallel, which was the case dealt with in Part I, then

$$\frac{\partial^2 h}{(\partial y')^2} = 0 \quad (2-9)$$

$$\frac{\partial^2 h}{\partial x' \partial y'} = 0 \quad (2-10)$$

2.2 Spatial Derivatives of the Wave Speeds. In this section relations are presented for connecting the geometric group speed G , the collinear group speed U , the phase speed v , and h . The water depth contours are assumed to be non-parallel.

a. Derivatives of v

The phase speed of a surface gravity water wave is defined (Part I)

$$v = \frac{1}{a} \tanh \frac{1}{2} \quad (2-11)$$

in which

$$a = \frac{2\pi}{gT} \quad (2-12)$$

$$b = \frac{T}{4\pi} \quad (2-13)$$

$$I = \frac{h}{bv} \quad (2-14)$$

where T is the wave period and g is the acceleration due to gravity.

The first partial derivatives of v in the $x'y'$ -coordinate system are given by

$$\frac{\partial v}{\partial x'} = W \frac{\partial h}{\partial x'} \quad (2-15)$$

$$\frac{\partial v}{\partial y'} = W \frac{\partial h}{\partial y'} = 0 \quad (2-16)$$

where

$$W = \frac{v(1 - a^2 v^2)}{[2abv^2 + h(1 - a^2 v^2)]} \quad (2-17)$$

The second partial derivatives of v are defined by

$$\frac{\partial^2 v}{(\partial x')^2} = W \left[\frac{\partial^2 h}{(\partial x')^2} + \gamma \left(\frac{\partial h}{\partial x'} \right)^2 \right] \quad (2-18)$$

$$\frac{\partial^2 v}{(\partial y')^2} = W \frac{\partial^2 h}{(\partial y')^2} \quad (2-19)$$

$$\frac{\partial^2 v}{\partial x' \partial y'} = W \frac{\partial^2 h}{\partial x' \partial y'} \quad (2-20)$$

where

$$\gamma = - \frac{4abv^2}{[2abv^2 + h(1 - a^2v^2)]^2} \quad (2-21)$$

b. Derivatives of U

The collinear group speed of a surface gravity water wave is defined (Part I)

$$U = \frac{1}{2} \left(1 + \frac{I}{\sinh I} \right) v \quad (2-22)$$

The first partial derivatives of U in the x'y'-coordinate system are given by

$$\frac{\partial U}{\partial x'} = \frac{\cosh I}{2} \left[(\sinh I + I) \frac{\partial v}{\partial x'} + v \frac{\partial I}{\partial x'} (1 - I \coth I) \right] \quad (2-23)$$

$$\frac{\partial U}{\partial y'} = \frac{\cosh I}{2} \left[(\sinh I + I) \frac{\partial v}{\partial y'} + v \frac{\partial I}{\partial y'} (1 - I \coth I) \right] = 0 \quad (2-24)$$

where

$$\frac{\partial I}{\partial x'} = I \left(\frac{1}{h} \frac{\partial h}{\partial x'} - \frac{1}{v} \frac{\partial v}{\partial x'} \right) \quad (2-25)$$

$$\frac{\partial I}{\partial y'} = I \left(\frac{1}{h} \frac{\partial h}{\partial y'} - \frac{1}{v} \frac{\partial v}{\partial y'} \right) = 0 \quad (2-26)$$

The second partial derivatives of U are given by

$$\begin{aligned} \frac{\partial^2 U}{(\partial x')^2} = & \frac{\cosh I}{2} \left[(\sinh I + I) \frac{\partial^2 v}{(\partial x')^2} + v \frac{\partial^2 I}{(\partial x')^2} (1 - I \coth I) \right] + \\ & + \frac{\partial I}{\partial x'} \left\{ -\frac{\partial v}{\partial x'} \coth I + \frac{\cosh I}{2} \left[\frac{\partial v}{\partial x'} (2 + \cosh I - I \coth I) + \right. \right. \\ & \left. \left. + v \frac{\partial I}{\partial x'} \cosh I (I \cosh I - \cosh I) \right] \right\} \end{aligned} \quad (2-27)$$

$$\frac{\partial^2 U}{(\partial y')^2} = \frac{\cosh I}{2} \left[(\sinh I + I) \frac{\partial^2 v}{(\partial y')^2} + v \frac{\partial^2 I}{(\partial y')^2} (1 - I \coth I) \right] \quad (2-28)$$

$$\frac{\partial^2 U}{\partial x' \partial y'} = \frac{\cosh I}{2} \left[(\sinh I + I) \frac{\partial^2 v}{\partial x' \partial y'} + v \frac{\partial^2 I}{\partial x' \partial y'} (1 - I \coth I) \right] \quad (2-29)$$

where

$$\begin{aligned} \frac{\partial^2 I}{(\partial x')^2} = & \frac{1}{I} \left(\frac{\partial I}{\partial x'} \right)^2 + I \left\{ \frac{1}{h} \left[\frac{\partial^2 h}{(\partial x')^2} - \frac{1}{h} \left(\frac{\partial h}{\partial x'} \right)^2 \right] - \right. \\ & \left. - \frac{1}{v} \left[\frac{\partial^2 v}{(\partial x')^2} - \frac{1}{v} \left(\frac{\partial v}{\partial x'} \right)^2 \right] \right\} \end{aligned} \quad (2-30)$$

$$\frac{\partial^2 I}{(\partial y')^2} = I \left(\frac{1}{h} \frac{\partial^2 h}{(\partial y')^2} - \frac{1}{v} \frac{\partial^2 v}{(\partial y')^2} \right) \quad (2-31)$$

$$\frac{\partial^2 I}{\partial x' \partial y'} = I \left(\frac{1}{h} \frac{\partial^2 h}{\partial x' \partial y'} - \frac{1}{v} \frac{\partial^2 v}{\partial x' \partial y'} \right) \quad (2-32)$$

c. Derivatives of G

The geometric group speed is defined by

$$G = U \cos \phi \quad (2-33)$$

where

$$\phi = \theta' - \gamma' \quad (2-34)$$

in which θ' is the direction of the wave packet, γ' is the direction of the wavelets, and both directions are measured with respect to the positive x' -axis. The first spatial derivatives of G are

$$\frac{\partial G}{\partial x'} = \frac{\partial U}{\partial x'} \cos \phi - U \sin \phi \frac{\partial \phi}{\partial x'} \quad (2-35)$$

$$\frac{\partial G}{\partial y'} = \frac{\partial U}{\partial y'} \cos \phi - U \sin \phi \frac{\partial \phi}{\partial y'} = 0 \quad (2-36)$$

The ray curvature for a wave packet is given by (Part I)

$$\frac{d\theta'}{ds_G} = \cos \theta' \frac{\partial \theta'}{\partial x'} + \sin \theta' \frac{\partial \theta'}{\partial y'} = \frac{1}{G} \left(\sin \theta' \frac{\partial G}{\partial x'} - \cos \theta' \frac{\partial G}{\partial y'} \right) \quad (2-37)$$

where ds_G is an element of arc length along the packet trajectory. Since the first derivatives in y' vanish it is found that

$$\frac{\partial \theta'}{\partial x'} = \frac{\tan \theta'}{G} \frac{\partial G}{\partial x'} \quad (2-38)$$

$$\frac{\partial \theta'}{\partial y'} = \frac{-\cot \theta'}{G} \frac{\partial G}{\partial y'} = 0 \quad (2-39)$$

In like manner, when considering the ray curvature of the wavelets it is found that

$$\frac{\partial \gamma'}{\partial x'} = \frac{\tan \gamma'}{v} \frac{\partial v}{\partial x'} \quad (2-40)$$

$$\frac{\partial Y'}{\partial y'} = - \frac{\cot Y'}{v} \frac{\partial v}{\partial y'} = 0 \quad (2-41)$$

Therefore

$$\frac{\partial \phi}{\partial x'} = \frac{\tan \theta'}{G} \frac{\partial G}{\partial x'} - \frac{\tan Y'}{v} \frac{\partial v}{\partial x'} \quad (2-42)$$

$$\frac{\partial \phi}{\partial y'} = - \frac{\cot \theta'}{G} \frac{\partial G}{\partial y'} + \frac{\cot Y'}{v} \frac{\partial v}{\partial y'} = 0 \quad (2-43)$$

When Equation (2-42) is substituted into (2-35) and the result is simplified it is found that

$$\frac{\partial G}{\partial x'} = \rho \left(\frac{\partial v}{\partial x'} \cos \phi + \sigma \frac{\partial v}{\partial x'} \right) \quad (2-44)$$

where

$$\rho = (1 + \tan \phi \tan \theta')^{-1} \quad (2-45)$$

$$\sigma = \frac{U}{v} \sin \phi \tan Y' \quad (2-46)$$

The second partial derivative of G with respect to x' is given by

$$\begin{aligned} \frac{\partial^2 G}{(\partial x')^2} = & \rho \left(\frac{\partial^2 v}{(\partial x')^2} \cos \phi + \sigma \frac{\partial^2 v}{(\partial x')^2} \right) + \left(\cos \phi \frac{\partial \rho}{\partial x'} - \right. \\ & \left. - \rho \sin \phi \frac{\partial \phi}{\partial x'} \right) \frac{\partial v}{\partial x'} + \left(\rho \frac{\partial \sigma}{\partial x'} + \sigma \frac{\partial \rho}{\partial x'} \right) \frac{\partial v}{\partial x'} \end{aligned} \quad (2-47)$$

where

$$\frac{\partial \rho}{\partial x'} = - \rho^2 \left(\tan \theta' \sec^2 \phi \frac{\partial \phi}{\partial x'} + \tan \phi \sec^2 \theta' \frac{\partial \theta'}{\partial x'} \right) \quad (2-48)$$

$$\frac{\partial G}{\partial x'} = \sigma \left(\frac{1}{U} \frac{\partial U}{\partial x'} - \frac{1}{v} \frac{\partial v}{\partial x'} \right) + \frac{U}{v} \left(\cos \phi \tan \gamma' \frac{\partial \phi}{\partial x'} + \sin \phi \sec^2 \gamma' \frac{\partial \gamma'}{\partial x'} \right) \quad (2-49)$$

The remaining second order spatial derivatives of G are obtained by differentiating Equation (2-36). Since the first derivatives in y' vanish it is found that

$$\frac{\partial^2 G}{(\partial y')^2} = \frac{\partial^2 U}{(\partial y')^2} \cos \phi - U \sin \phi \frac{\partial^2 \phi}{(\partial y')^2} \quad (2-50)$$

$$\frac{\partial^2 G}{\partial x' \partial y'} = \frac{\partial^2 U}{\partial x' \partial y'} \cos \phi - U \sin \phi \frac{\partial^2 \phi}{\partial x' \partial y'} \quad (2-51)$$

The second partial derivative of ϕ with respect to y' is found by differentiating Equation (2-43). The result is

$$\frac{\partial^2 \phi}{(\partial y')^2} = - \frac{\cot \theta'}{G} \frac{\partial^2 G}{(\partial y')^2} + \frac{\cot \gamma'}{v} \frac{\partial^2 v}{(\partial y')^2} \quad (2-52)$$

When Equation (2-52) is substituted into (2-50) and the terms rearranged it is found that

$$\frac{\partial^2 G}{(\partial y')^2} = \zeta \left(\frac{\partial^2 U}{(\partial y')^2} \cos \phi - \xi \frac{\partial^2 v}{(\partial y')^2} \right) \quad (2-53)$$

where

$$\zeta = (1 - \tan \phi \cot \theta')^{-1} \quad (2-54)$$

$$\xi = \frac{U}{v} \sin \phi \cot \gamma' \quad (2-55)$$

After differentiating Equation (2-43) with respect to x' it is found that

$$\frac{\partial^2 \phi}{\partial x' \partial y'} = \frac{\tan \theta'}{G} \frac{\partial^2 G}{\partial x' \partial y'} - \frac{\tan \gamma'}{v} \frac{\partial^2 v}{\partial x' \partial y'} \quad (2-56)$$

When Equation (2-56) is substituted into (2-51) the simplified result becomes

$$\frac{\partial^2 G}{\partial x' \partial y'} = \rho \left(\frac{\partial^2 U}{\partial x' \partial y'} \cos \phi + \sigma \frac{\partial^2 v}{\partial x' \partial y'} \right) \quad (2-57)$$

2.3 Ray Curvature for Nonparallel Water Depth Contours. The ray curvature κ_G depends only on derivatives of the first order. By making the calculations in the $x'y'$ -coordinate system the first derivatives in y' vanish. This results in a simplified ray curvature expression which is formally the same as presented in Part I for locally parallel water depth contours.

$$\kappa_G = \frac{\sin \theta'}{G} \frac{\partial G}{\partial x'} \quad (2-58)$$

2.4 Ray Separation Equation for Nonparallel Water Depth Contours. The ray separation equation is defined (Part I)

$$\frac{d^2 \beta}{dt^2} + p \frac{d\beta}{dt} + q\beta = 0 \quad (2-59)$$

where β is the ray separation factor, t is time, and

$$p = -2 \cos \theta' \frac{\partial G}{\partial x'} \quad (2-60)$$

$$q = G \left(\sin^2 \theta' \frac{\partial^2 G}{(\partial x')^2} - 2 \sin \theta' \cos \theta' \frac{\partial^2 G}{\partial x' \partial y'} + \cos^2 \theta' \frac{\partial^2 G}{(\partial y')^2} \right) \quad (2-61)$$

In the $x'y'$ -coordinate system p is simplified as is κ_G . In q there is also a reduction in the expressions for the second order derivatives involving y' .

CHAPTER III SUBROUTINE SURFCE

3.1 Modifications to Subroutine SURFCE. Modifications have been made to the program subroutine SURFCE in Part I in order to remove the restriction that the water depth contours be locally parallel about each ray point and to simplify some of the computations. When the program is run with these modifications minor changes will occur in the printed output for the sample input data presented in Part I.

Referring to the program listing for SURFCE on pages 52-54 of Part I the following modifications have been made. In place of SURFACE 58 an alternative expression is used to compute $\partial h / \partial x'$ (DHDX). Between SURFACE 87 and SURFACE 88 $\partial I / \partial x'$ (DIDX) is computed as are the hyperbolic sine, cosine, and tangent of I. Statements SURFACE 91 through SURFACE 93 for computing $\partial U / \partial x'$ (DUDX) have been replaced by a simplified expression. Between SURFACE 107 and SURFACE 108 $\partial^2 h / (\partial y')^2$ (DHDYY) and $\partial^2 h / \partial x' \partial y'$ (DHDXY) are computed. SURFACE 111 has been replaced by expressions to compute $\partial^2 v / (\partial y')^2$ (DVDYY) and $\partial^2 v / \partial x' \partial y'$ (DVDXY). Statements SURFACE 114 through SURFACE 120 have been replaced by expressions for calculating $\partial^2 I / (\partial y')^2$ (DIDYY) and $\partial^2 I / \partial x' \partial y'$ (DIDXY). A simplified expression has been substituted for $\partial^2 U / (\partial x')^2$ (DUDXX). Also, the values of $\partial^2 U / (\partial y')^2$ (DUDYY) and $\partial^2 U / \partial x' \partial y'$ (DUDXY) are computed. Between SURFACE 122 and SURFACE 123 ζ (ZETA), ξ (XI), $\partial^2 G / (\partial y')^2$ (DGDYY), and $\partial^2 G / \partial x' \partial y'$ (DGDXY) are determined. Simplified expressions are used to compute $\partial^2 G / (\partial y')^2$ and $\partial^2 G / \partial x' \partial y'$ when $|\tan \theta'| \leq \tan 5^\circ$ and $|\tan \gamma'| \leq \tan 5^\circ$. Statement SURFACE 124 has been replaced by the complete expression for computing q (QOT).

3.2 Description of Subroutine SURFCE. SURFCE is called by RAYN and MOVE to calculate h , α , γ , G , p , q , k_g , and other ray particulars. At the first ray point twelve values of h from CMAT are selected about the point as shown in Figure (2-2) of Part I. A quadratic surface is fit to the set of water depths. At successive ray points the quadratic surface is determined only if there is a change in the set of twelve water depths. The water depth and its partial derivatives in the fixed xy -system, $\partial h / \partial x$, $\partial h / \partial y$, $\partial^2 h / \partial x^2$, $\partial^2 h / \partial y^2$, and $\partial^2 h / \partial x \partial y$, are determined at the ray point by interpolating on the quadratic surface.

If $h \leq 0$, NDP = 2 and there is a RETURN. If $h > 0$ the ratio of the water depth to the deep water wavelength is computed. If $h / \lambda_d > 0.64$, which defines deep water, NFK = 1. If $h / \lambda_d \leq 0.64$, NFK = 2. VELCTY is called, and after the return if NFK = 1, W = 0. If NFK = 2, CONDER is called to

compute W. The values of $\partial v/\partial x$ and $\partial v/\partial y$ are calculated using W.

At each ray point calculations are made in a $x'y'$ -coordinate system which is chosen with the positive x' -axis in the direction of the gradient of the water depth. The value of $\partial h/\partial x'$ is computed, and if it exceeds 0.00001 the angle α by which the x' -axis is rotated with respect to the x -axis is computed. If $|\partial h/\partial x'| \leq 0.00001$ the water depth is assumed to be constant and α remains constant.

If FLAG1 = 0, γ' is computed, and if necessary it is placed within the range $|\gamma'| \leq 360^\circ$. A check is made to determine if there is total reflection. If there is, FLAG2 = 1 and there is a RETURN. Otherwise, FLAG2 = 0 and the new γ' is computed using Snell's law with phase velocity following a set of rules. Using the values of γ' , γ is computed. When FLAG 1 \neq 0 these steps for computing the new wavelet direction and the test for total reflection are omitted.

The values of ϕ , G, and $\partial v/\partial x'$ are calculated. If NFK = 2, $\partial U/\partial x'$ is determined using its unsimplified expression. If NFK \neq 2, the deep water formula is used to calculate $\partial U/\partial x'$. The value of $\partial U/\partial x'$ is used in computing $\partial G/\partial x'$.

If NFK \neq 2, the coefficients of the ray separation equation and the ray curvature are set equal to zero. Then there is a RETURN. If NFK = 2, p , $\partial^2 h/(\partial x')^2$, $\partial^2 h/(\partial y')^2$, $\partial^2 h/\partial x'\partial y'$, $\partial^2 v/(\partial x')^2$, $\partial^2 v/(\partial y')^2$, $\partial^2 v/\partial x'\partial y'$, $\partial^2 U/(\partial x')^2$, $\partial^2 U/(\partial y')^2$, $\partial^2 U/\partial x'\partial y'$, $\partial^2 G/(\partial x')^2$, $\partial^2 G/(\partial y')^2$, $\partial^2 G/(\partial x'\partial y')$, q , and κ_G are computed. This is followed by a return.

3.3 Listing of Subroutine SURFCE

SUBROUTINE SURFCE (X,Y,A,FK,NFK,NDP,AV)	SURFACE	1
DIMENSION S(6,F),EM(6,12),C(12),YVW(6),E(6)	SURFACE	2
\$,CMAT(100,100),AA(2000),AY(2000),CONTUR(9)	SURFACE	3
REAL KR,KF,KS,KPTOL,KFC	SURFACE	4
INTEGER FLAG1,FLAG2	SURFACE	5
COMMON S,EM,E,YVW,CMAT,C,AX,AY,CUNTUR,PROJCT,GRID,DCON,FAN,DATE1	SURFACE	6
\$,DATE2,CIN,DIR,ROP,TT,WBCOP,MOE,DY,DELTAT,SDLTAT,D,HGT,HGTZ,SVX	SURFACE	7
\$,SVY,SOEP,W,DEF,WL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVG,DUD,KS	SURFACE	8
\$,DGD,SV,SA,TPI,SAV,SVAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,UHDX	SURFACE	9
\$,SVFKB,SAVFK,FKEAR,MAXQ,SK,FLC,NUMT,INUM,IFLG,RCOUNT,AMM,ANN	SURFACE	10
\$,REFLECT,RELBUM,REFPCT,REFRUM,BRK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,CF,BZ	SURFACE	11
\$,SBZ,RDZ,SDNZ,KRTOL,KK,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5	SURFACE	12
C IN THIS SUBROUTINE THE WATER DEPTH, POTATTON ANGLE, HAVELET	SURFACE	13
C DIRECTION, GEOMETRIC GROUP VELOCITY, COEFFICIENTS OF THE RAY	SURFACE	14
C SEPARATION EQUATION, AND THE PACKET RAY CURVATURE ARE COMPUTED.	SURFACE	15
C SUBROUTINES VELCTY AND CONDER ARE CALLED.	SURFACE	16
I=X \$ J=Y \$ FI=1 \$ FJ=J \$ XL=X+1.-FI \$ YL=Y+1.-FJ	SURFACE	17
IF (MAXQ.LE.1) GO TO 1	SURFACE	18
IF (ZI.NE.FI) GO TO 1	SURFACE	19
IF (ZJ.EQ.FJ) GO TO 3	SURFACE	20
1 ZI=FI \$ ZJ=FJ	SURFACE	21
C SELECT 12 WATER DEPTHS ABOUT RAY POINT	SURFACE	22
C(1)=CMAT(J+1,1) \$ C(2)=CMAT(J+2,1) \$ C(3)=CMAT(J,I+1)	SURFACE	23
C(4)=CMAT(J+1,I+1) \$ C(5)=CMAT(J+2,I+1) \$ C(6)=CMAT(J+3,I+1)	SURFACE	24
C(7)=CMAT(J,I+2) \$ C(8)=CMAT(J+1,I+2) \$ C(9)=CMAT(J+2,I+2)	SURFACE	25
C(10)=CMAT(J+3,I+2) \$ C(11)=CMAT(J+1,I+3) \$ C(12)=CMAT(J+2,I+3)	SURFACE	26
C FIT QUADRATIC SURFACE TO 12 WATER DEPTHS	SURFACE	27
DO 318 II=1,6	SURFACE	28
YVW(II)=0.	SURFACE	29
DO 318 L=1,12	SURFACE	30
YVW(II)=YVW(II)+C(L)*EM(II,L)	SURFACE	31
318 CONTINUE	SURFACE	32
DO 319 JJ=1,6	SURFACE	33
E(II)=0.	SURFACE	34
DO 319 JJ=1,6	SURFACE	35
E(II)=E(II)+S(JJ,II)*YVW(JJ)	SURFACE	36
319 CONTINUE	SURFACE	37
C COMPUTE INTERPOLATED WATER DEPTH	SURFACE	38
3 DEP=(E(1)+E(2)*XL+E(3)*YL+E(4)*XL**2+E(5)*XL*YL+E(6)*YL**2)*DCON	SURFACE	39
C COMPUTE PARTIAL DERIVATIVES OF WATER DEPTH IN FIXED XY-SYSTEM	SURFACE	40
HX=(E(2)+2.*E(4)*XL+E(5)*YL)*DCON	SURFACE	41
HY=(E(3)+E(5)*XL+E(6)*2.*YL)*DCON	SURFACE	42
HXX=2.*E(4)*DCON \$ HYY=2.*E(6)*DCON \$ HXY=E(5)*DCON	SURFACE	43
IF (DEP.GT.0.) GO TO 324	SURFACE	44
NDP=2	SURFACE	45
GO TO 403	SURFACE	46
324 IF (DEP/WL.GT..64) GO TO 322	SURFACE	47
NFK=2	SURFACE	48
GO TO 323	SURFACE	49
322 NFK=1	SURFACE	50
323 CALL VELCTY(V,TT,MAXQ,DEP,NFK,U)	SURFACE	51
IF (NFK.EQ.2) GO TO 402	SURFACE	52
W=0.	SURFACE	53
GO TO 10	SURFACE	54
402 CN=1.	SURFACE	55
CALL CONDER(DN,TT,V,MAXQ,NFK)	SURFACE	56
W=DN	SURFACE	57


```

10  VX=W*HX $ VY=W*HY $ DHDX=HX*COS(ALFA)+HY*SIN(ALFA)
    IF (ABS(DHDX/GRID) .GT. 0.00001) GO TO 8
    GO TO 9
C COMPUTE ROTATION ANGLE
8  ALFA=ATAN2(HY, HX)
9  IF (FLAG1 .NE. 0) GO TO 12
C COMPUTE WAVELET DIRECTION IN ROTATED XY-SYSTEM USING SNELLS LAW
C WITH V AND TEST FOR TOTAL REFLECTION DUE TO THE WAVELETS
    GP=SAV-ALFA
14  IF (ABS(GP) .LE. 6.2831853) GO TO 13
    IF (GP) 16, 13, 17
16  GP=GP+6.2831853
    GO TO 14
17  GP=GP-6.2831853
    GO TO 14
13  ARG1=V*SIN(GP)/SAVV
    IF (ABS(ARG1) .LE. 1.) GO TO 18
    FLAG2=1.
    GO TO 403
18  FLAG2=0. $ GPT=ASIN(ARG1)
    IF (ABS(GP) .LE. 4.7123889) GO TO 20
    AVP=6.2831853+GPT
    GO TO 22
20  IF (ABS(GP) .LE. 1.5707963) GO TO 23
    AVP=3.1415927-GPT
    GO TO 22
23  AVP=GPT
22  AV=AVP+ALFA
12  PHI=A-AV $ G=U*COS(PHI)
    DVOX=W*DHDX $ BAR3=12.5663708/TI $ BAR4=BAR3*DEP/V
    DIDX=BAR4*(DHDX/DEP-DVOX/V) $ SINHI=(EXP(BAR4)-EXP(-BAR4))/2.
    COSHI=(EXP(BAR4)+EXP(-BAR4))/2. $ TANHI=SINHI/COSHI
    IF (INFK .EQ. 2) GO TO 25
    DUOX=DVOX/2.
    GO TO 27
25  DUOX=(.5/SINHI)*((SINHI+BAR4)*DVOX+V*DIDX*(1.-BAR4/TANHI))
27  RHO=1./(1.+TAN(PHI)*TAN(A-ALFA)) $ SIGMA=U*SIN(PHI)*TAN(AV-ALFA)/V
    DGOX=RHO*(DUOX*COS(PHI)+SIGMA*DVOX)
    IF (INFK .EQ. 2) GO TO 28
    POT=0. $ QOT=). $ FK=0.
    GO TO 403
C COMPUTE P IN ROTATED XY-SYSTEM
28  POT=-2.*COS(A-ALFA)*DGOX/GRID $ DAVDX=TAN(AV-ALFA)*DVOX/V
    DAOX=TAN(A-ALFA)*DGOX/G $ DPHIDX=DAOX-DAVOX
    DRHODX=-(RHO**2)*(TAN(A-ALFA)*DPHIDX/(COS(PHI)**2)+
    $ TAN(PHI)*DAOX/(COS(A-ALFA)**2))
    CSIGDX=SIGMA*(DUOX/U-DVOX/V)+U*(COS(PHI)*TAN(AV-ALFA)*DPHIDX+
    $ SIN(PHI)*DAOX/(COS(AV-ALFA)**2))/V
    DHDXX=(COS(ALFA)**2)*HXX+2.*SIN(ALFA)*COS(ALFA)*HXY+
    $ 2.*HYY
    DHYY=(SIN(ALFA)**2)*HXX-2.*SIN(ALFA)*COS(ALFA)*HXY
    $ +(COS(ALFA)**2)*HYY
    DHOXY=(SIN(ALFA)*COS(ALFA))*((HYY**2)-(HXX**2))+
    $ ((COS(ALFA)**2)-(SIN(ALFA)**2))*HXY
    SMA=6.2831854/(32.2*TI) $ SMAB=1./64.4
    DVOXX=W*(DHDXX+(DHXX**2)*(-4.*SMAB*(V**2)/((2.*SMAB*(V**2)+DEP*
    $ (1.-(SMA*V)**2))**2)))
    DVOYY=W*DHYY $ DVOXY=W*DHDX

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16

	CIDXX=(DIOX**2)/BAR4+BAR4*((CHDX-(DHOX**2)/DEP)/DEP-(DVOXX	SURFACE	116
	\$-(DVOXX**2)/V)/V)	SURFACE	117
	DIDYY=BAR4*(DHOXY/LEF-DVCYY/V) \$ CIOXY=BAR4*(DHOXY/DEP-DVOXY/V)	SURFACE	118
30	DUDXX=(.5/SINH I)*(-(SINH I+BAR4)*DVOX+V*DIOX*(1.-BAR4/TANH I))*DIOX	SURFACE	119
	\$/TANH I*(SINH I+BAR4)*DVOXX+DVOX*DIOX*(2.+COSH I-BAR4/TANH I)+V*(CIEXX	SURFACE	120
	\$*(1.-BAR4/TANH I)+(CIOX**2)*(BAR4/SINH I-COSH I)/SINH I)	SURFACE	121
	DUDYY=(.5/SINH I)*((SINH I+BAR4)*DVOYY+V*DIOY*(1.-BAR4/TANH I))	SURFACE	122
	DUDXY=(.5/SINH I)*((SINH I+BAR4)*DVOXY+V*OIOY*(1.-BAR4/TANH I))	SURFACE	123
32	DGDXX=RHO*(COS (PHI)*CLOXX+SIGMA*DVOXX)+(CUS (PHI)*DRHOX-RHO*SIN	SURFACE	124
	\$(PHI)*DPHI CX)*CUDX+(RHO*DSIGX+SIGMA*DRHOX)*DVOX	SURFACE	125
	IF (ABS(TAN(A-ALFA))) .GT. 0.08748866 .AND. ABS(TAN(AV-ALFA)) .GT.	SURFACE	126
	\$0.06748866) GO TO 35	SURFACE	127
	ZETA=1. \$ XI=0.	SURFACE	128
	GO TO 36	SURFACE	129
35	ZETA=1.0/(1.0-TAN(PHI)/TAN(A-ALFA)) \$ XI=U*SIN(PHI)/V*TAN(AV-ALFA)	SURFACE	130
36	DGDYY=ZETA*(DUCYY*COS(PHI)-XI*DVCYY)	SURFACE	131
	DGDXY=RHO*(DUDXY*CCS(PHI)+SIGMA*DVOXY)	SURFACE	132
C	COMPUTE Q IN ROTATED XY-SYSTEM	SURFACE	133
	QOT=(G*((SIN(A-ALFA)**2)*DGDXX-2.*SIN(A-ALFA)*COS(A-ALFA)*DGDXY+	SURFACE	134
	\$(COS(A-ALFA)**2)*DGDYY))/(GRIC**2)	SURFACE	135
C	COMPUTE PACKET RAY CURVATURE IN ROTATED XY-SYSTEM	SURFACE	136
	FK=SIN(A-ALFA)*UGDX/G	SURFACE	137
403	RETURN	SURFACE	138
	END	SURFACE	139

CHAPTER IV WAVE PREDICTION EXAMPLES FOR NONPARALLEL WATER DEPTH CONTOURS

Two examples of nonparallel water depth contours are used to illustrate the wave prediction program.

4.1 Sinusoidal to Parallel Water Depth Contours. Figure (4-1) shows a set of wave packet trajectories for a wave period of 7.0 seconds. The rays begin in deep water (initial water depth equal to or greater than one half the wavelength). The water depth contours are sinusoidal at the shoreline and gradually become parallel at a water depth of 38.6 meters. The amplitude of the water depth contour at the shoreline is 2.5 kilometers, and the contour wavelength is 10 kilometers. GRID, the distance between grid points, is 156.25 meters.

From the figure it is seen that the energy is fairly evenly distributed along the coastline with slightly more energy at the headland than in the bay. Figures (4-2) through (4-4) show the printed output for rays number 4, 8, and 16, respectively, of Figure (4-1). The computed refraction coefficients are in good agreement with values estimated from the plot.

4.2 Sinusoidal Water Depth Contours. In Figure (4-5) the period of the wave packet trajectories is 14.0 seconds and they begin in deep water. The water depth contours are sinusoidal with an amplitude of 5 kilometers and a wavelength of 20 kilometers. GRID has a value of 312.5 meters.

This example is quite interesting since there is decidedly more energy in the bay than at the headland. The opposite result would be expected for monochromatic trajectories. The refraction of wave packets could explain why there is more erosion in bays than at headlands for some coastlines. Figures (4-6) through (4-8) show the printed output for rays number 4, 8, and 16, respectively, of Figure (4-5). The computed refraction coefficients agree favorably with values estimated from the plot.

PROJ. NO. SIP 2, 78/11/13
 SCL = 1/98425, CIN = 0
 PLOT NO. 1, DIR. = WAVPAK

SOUNDINGS IN METERS

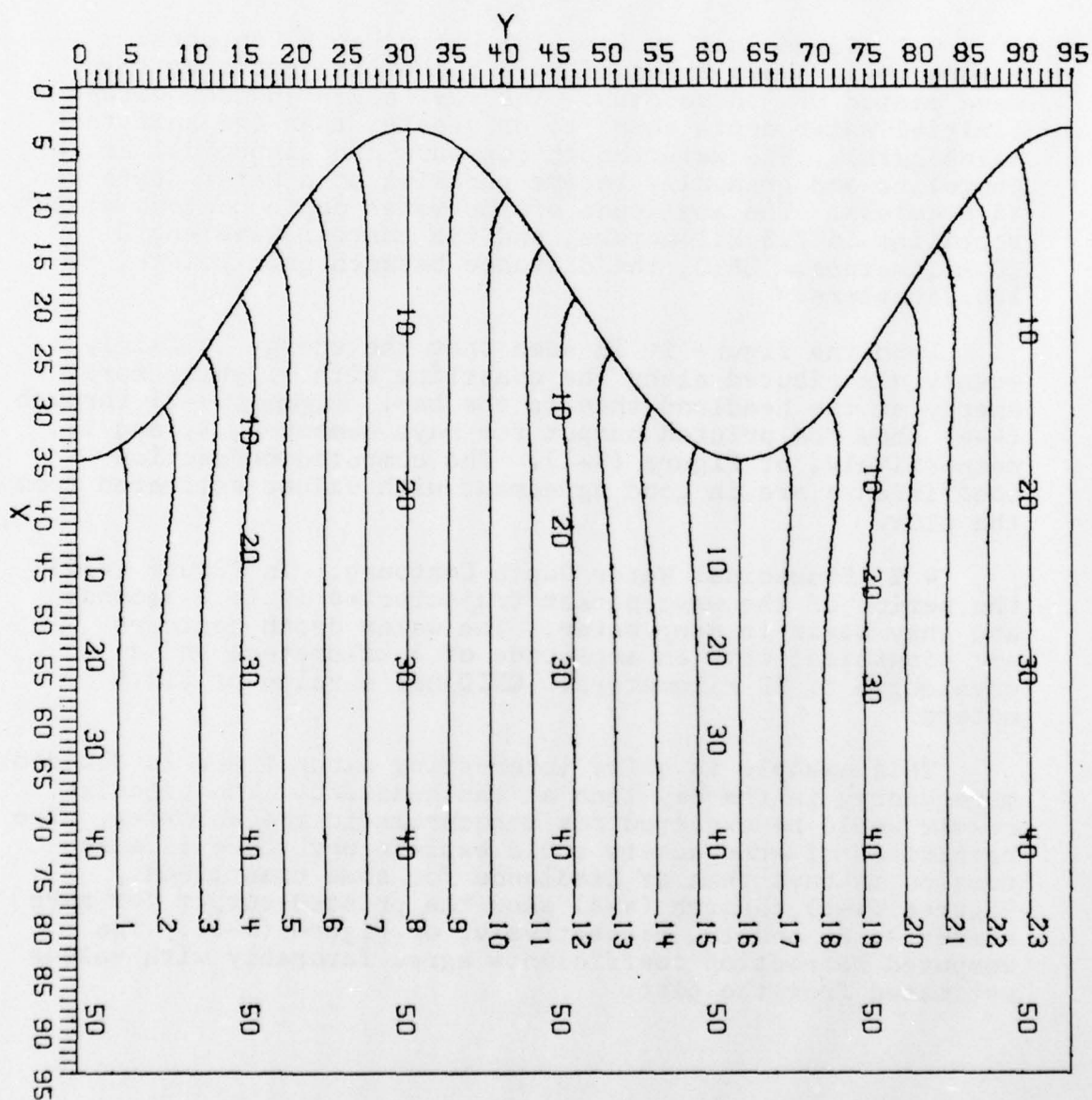


Figure (4-1). Plot for sinusoidal to parallel water depth contours.

PROJECT NO. STP 2, 78/11/13, PLOT NO. 1, PERIOD= 7.0SEC., RAY NO. 4, DELTAY= 10.00, CF=0.000000, KRTOI= .001000

THE OUTPUT IS IN METRIC UNITS. H,HGT(METER). G,U,V(METER/SECOND).
D(BETA)/DT = -1.045E-11

[illegible]

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Figure (4-2). Printed output for Ray No. 4 in Figure (4-1).

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PROJECT NO. SIP 2, 70/11/13, PLOT NO. 1, PERIOD= 7.0SEC., RAY NO. 0, DELTAT= 10.00, CF=0.000000, KRTOL= .001000
```

THE OUTPUT IS IN METRIC UNITS. H, HGT(METER). G, U, V(METER/SECOND).

$$C(BETA)/DT = -1.834E-11$$

MAX X	Y	H	PACK	WAVE	D	FK	ALFA	G	U	V	HGT	KS	KF	NO KR	PCTDIF
-------	---	---	------	------	---	----	------	---	---	---	-----	----	----	-------	--------

[illegible]

Figure (4-3). Printed Output for Ray No. 8 in Figure (4-1).

PROJ. NO. SIN 3, 78/08/31
SCL = 1/196850, CIN = 0
PLOT NO. 1, DIR. = WAVPAK

SOUNDINGS IN METERS

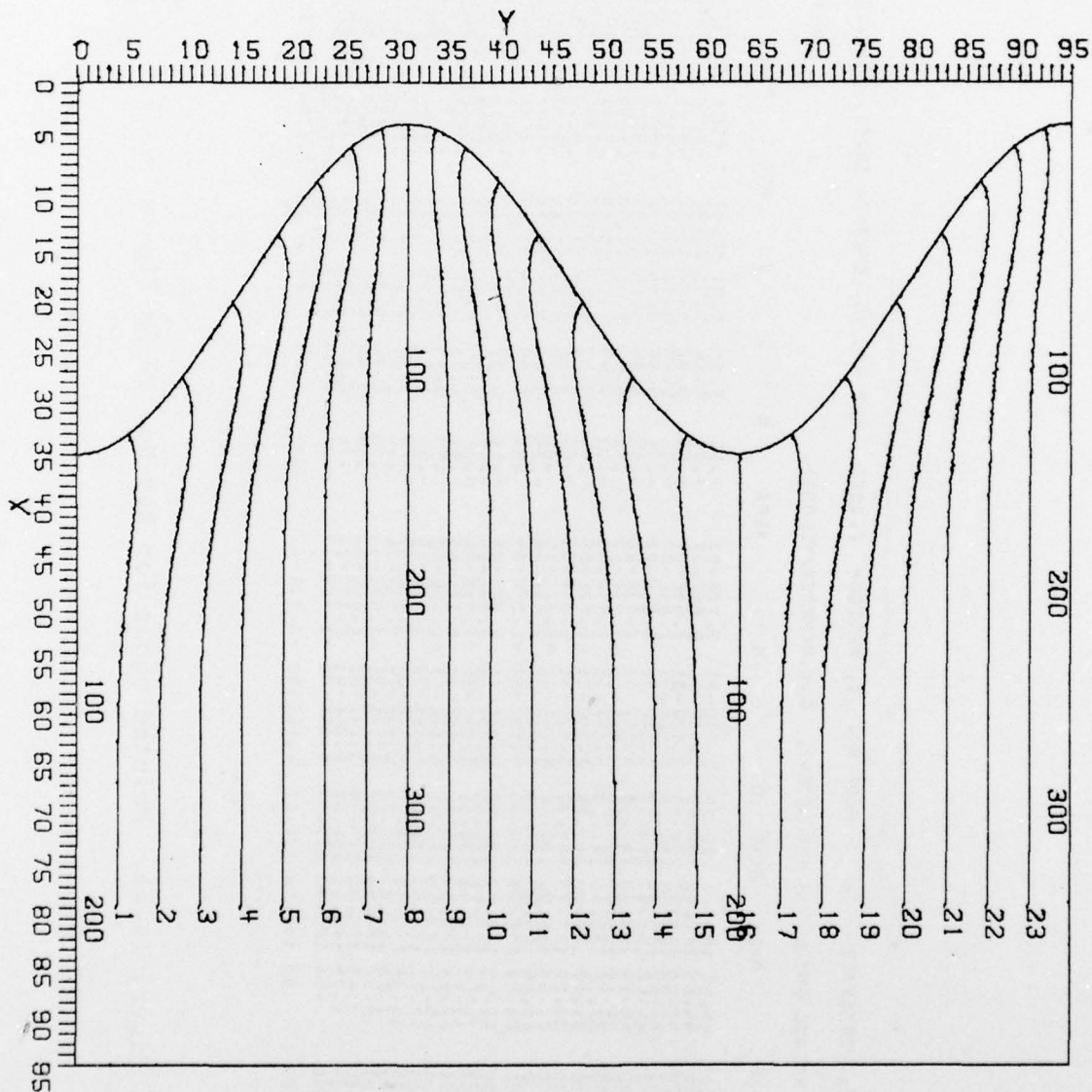


Figure (4-5). Plot for sinusoidal water depth contours.

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PROJECT NO. SIN 3, 78/08/31 , PLOT NO. 1, PERIOD= 14.0SEC., RAY NO. 8, CELTAT= 10.00, CF=0.000000, KATOL= .001000

THE OUTPUT IS IN METRIC UNITS. H,HGT(METER). G,U,V(METER/SECOND).

$$D(\text{BETA})/DT = -0.$$
[illegible]

Figure (4-7). Printed output for Ray No. 8 in Figure (4-5).

PROJECT NO. SIN 3, 78/08/31 , PLOT NO. 1, PERIOD= 14.0SEC., RAY NO. 16, BELTAT= 10.00, CF=0.000000, KRTOL= .001000

THE OUTPUT IS IN METRIC UNITS. H,HGT(METER). G,U,V(METER/SECOND). D(BETA)/DT = -0.

MAX X	Y	H	PACK	WAVE	D	FK	ALFA	G	U	V	HGT	KS	KF	NO KR	PCTDIF
1	80.00	64.00	209.25	180.00	180.00	0.00	15	10.93	9.33	10.93	1.00	1.00	1.00	1.00	0.00
105	76.85	64.00	191.47	180.00	180.00	-6.90E-01	12	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
205	75.36	64.00	183.23	180.00	180.00	-0.07	05	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
305	73.78	64.00	174.65	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
405	69.60	64.00	150.37	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
505	65.21	64.00	141.01	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
605	61.59	64.00	133.13	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
705	57.55	64.00	124.65	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
805	53.89	64.00	116.39	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
905	50.99	64.00	108.33	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
1005	47.93	64.00	100.56	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
1105	44.87	64.00	93.21	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
1205	42.77	64.00	86.48	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
1305	40.81	64.00	80.28	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
1405	39.01	64.00	74.64	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
1505	37.58	64.00	69.54	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
1605	36.48	64.00	64.98	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
1705	35.65	64.00	60.94	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
1805	35.05	64.00	57.41	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
1905	34.68	64.00	54.38	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
2005	34.48	64.00	51.84	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
2105	34.40	64.00	49.78	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
2205	34.40	64.00	48.18	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
2305	34.40	64.00	46.98	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
2405	34.40	64.00	46.14	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
2505	34.40	64.00	45.62	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
2605	34.40	64.00	45.36	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
2705	34.40	64.00	45.28	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
2805	34.40	64.00	45.32	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
2905	34.40	64.00	45.40	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
3005	34.40	64.00	45.52	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
3105	34.40	64.00	45.68	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
3205	34.40	64.00	45.88	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
3305	34.40	64.00	46.12	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
3405	34.40	64.00	46.40	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
3505	34.40	64.00	46.72	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
3605	34.40	64.00	47.08	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
3705	34.40	64.00	47.48	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
3805	34.40	64.00	47.92	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
3905	34.40	64.00	48.40	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
4005	34.40	64.00	48.92	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
4105	34.40	64.00	49.48	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
4205	34.40	64.00	50.08	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
4305	34.40	64.00	50.72	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
4405	34.40	64.00	51.40	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
4505	34.40	64.00	52.12	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
4605	34.40	64.00	52.88	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
4705	34.40	64.00	53.68	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
4805	34.40	64.00	54.52	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
4905	34.40	64.00	55.40	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
5005	34.40	64.00	56.32	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
5105	34.40	64.00	57.28	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
5205	34.40	64.00	58.28	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
5305	34.40	64.00	59.32	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
5405	34.40	64.00	60.40	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
5505	34.40	64.00	61.52	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
5605	34.40	64.00	62.68	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
5705	34.40	64.00	63.88	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
5805	34.40	64.00	65.12	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
5905	34.40	64.00	66.40	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
6005	34.40	64.00	67.72	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
6105	34.40	64.00	69.08	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
6205	34.40	64.00	70.48	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
6305	34.40	64.00	71.92	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
6405	34.40	64.00	73.40	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
6505	34.40	64.00	74.92	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
6605	34.40	64.00	76.48	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
6705	34.40	64.00	78.08	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
6805	34.40	64.00	79.72	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
6905	34.40	64.00	81.40	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
7005	34.40	64.00	83.12	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
7105	34.40	64.00	84.88	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
7205	34.40	64.00	86.68	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
7305	34.40	64.00	88.52	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
7405	34.40	64.00	90.40	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
7505	34.40	64.00	92.32	180.00	180.00	-0.06	03	10.93	10.93	10.93	1.00	1.00	1.00	1.00	0.00
7605	34.40	64.00	94.28	180.00	180.00										

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4. TITLE (and Subtitle) A Method for Calculating Wave Packet Trajectories and Wave Heights: Part II		5. TYPE OF REPORT & PERIOD COVERED Technical
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) J. Ernest Breeding, Jr.		8. CONTRACT OR GRANT NUMBER(s) N00014-77-C-0329 <i>new</i>
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Oceanography The Florida State University Tallahassee, FL 32306		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR388-138
11. CONTROLLING OFFICE NAME AND ADDRESS Geography Programs, Department of the Navy Office of Naval Research Arlington, VA		12. REPORT DATE November, 1978
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for Public Release; Distribution Unlimited.		
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19. KEY WORDS (Continue on reverse side if necessary and identify by block number) <div style="display: flex; justify-content: space-between;"> <div> wave packets wave refraction packet ray curvature </div> <div> wave height refraction coefficient geometric group velocity </div> </div>		
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Block 20: ABSTRACT



In Part I a method for calculating wave packet trajectories and wave heights is based on the assumption that the water depth contours are locally parallel in the vicinity of each ray point. This method is extended in order to predict the modification to surface gravity water waves in shoaling water when the water depth contours are not parallel. The calculations are greatly simplified by choosing a coordinate system at each ray point in which one axis is aligned parallel with the direction of the gradient of the water depth. Example printouts and plots are presented to illustrate the wave prediction method. It is discovered that when waves initially approach sinuous water depth contours symmetrically with respect to the beach there can be more energy in the bays than at the headlands.

